KMR RESEARCH SEMINAR ON GEOMETRIC GROUP THEORY 2014
AGOL'S THEOREM ON SPECIAL CUBE COMPLEXES.

The talks will be 60 minutes + 15 minutes for questions/discussion. Good introductions to Agol's Theorem and its proof are for instance [7] and [9]. Basically the only prerequisite for the seminar is familiarity with the definition of hyperbolic groups and CAT(0) spaces, as can be found for example in [8].

1. Special Cube Complexes and Haglund-Wise Theory

Talk 1: Introduction to cube complexes.
Given an overview of cube complexes. In particular, explain the notion of non-positive curvature for cube complexes and Gromov's link criterion. Hyperplanes. This should cover roughly the material of the first four chapters in Petra's lecture notes, but can be based on other sources as well.
Literature: For instance: [18], [17], [19].

Talk 2: Right Angled Artin Groups.
Introduce Coxeter groups and right-angled Artin groups. Examples and basic properties. Explain their relation to cube complexes. RAAGs are linear.
Literature: For instance [10], [11].

Talk 3: Special Cube Complexes I: Introduction.
Introduce special cube complexes and some of their properties needed in the proof of Haglund-Wise. Hyperplanes. Cube completion. Examples of special cube complexes. If time permits, relate different notions of "special" [14, Proposition 3.10].
Literature: [14].

Talk 4: Special Cube Complexes II: Haglund-Wise Theory.
Special groups embed into RAAGs: [14, Theorem 4.4]. The Separability Theorem [14, Theorem 7.3] (in detail), [14, Theorem 8.13 and 8.14].
Literature: [14].

Talk 5: Weak Separation Theorem (Agol-Groves-Manings).
Present the Weak Separation Theorem [1, Theorem A.1], and give an overview of the proof.
Literature: [1, Appendix A], [3].

2. 3-manifolds and cubulations

Talk 6: Sageev's cubulation construction
Codimension 1-subgroups. Sageev's cubulation construction [15, Proposition 3.1], in detail. Examples of the construction. Cocompactness in the hyperbolic case [16, Proposition 3.1].
Literature: [15], [16], [7].

Talk 7: Overview on 3-manifolds
Explain basic properties of 3-manifolds. The Prime Decomposition Theorem and the Perelman-Thurston Geometrisation Theorem. Examples. Separability and lifting of immersed surfaces. This talk will be 75 + 15 minutes.
Literature: [4]

Date: January 10, 2014.

Talk 8: Cubulating hyperbolic fundamental groups.
Give a short overview of the work of Kahn-Markovic and explain the main result [12, Theorem 1.1] [6, Proposition 5.1]. Discuss in detail the boundary criterion for cubulation [6, Theorem 1.4], and the corollary that fundamental groups of hyperbolic manifolds are cubulated [6, Theorem 5.3].
Literature: [5], [6], [12].

3. Agol's Theorem and Applications

Talk 9: Agol's theorem, the Virtual Haken Conjecture and other applications.
Present Agol's theorem and give a very short overview of the proof. Prove the Virtual Haken Conjecture. Give some applications of this conjecture. If time permits, indicate other applications to Geometric Group Theory and Geometry.
Literature: [1], [2], [9, Section 8].

Talk 10: The Virtual Fibration Conjecture. Explain and prove Agol's Virtual Fibration Criterion. Deduce the Virtual Fibration Conjecture.
Literature: [2], [13]

Talk 11: Proof of Agol's Theorem I: The quasi-convex virtual hierarchy.
Introduce the fundamental definitions for the proof, including the notion of quasi-convex virtual hierarchy. Virtual Gluing. Quotients with compact walls.
Literature:[1, Section 2, 3, 4], [10].

Talk 12: Proof of Agol's Theorem II: Invariant coloring measures.
Prove [1, Theorem 5.2]. Discuss cube complexes with boundary patterns.
Literature: [1, Section 5, 6], [10].

Talk 13: Proof of Agol's Theorem III: Virtually gluing up the hierarchy.
Finish the proof of Agol's Theorem.
Literature: [1, Section 8], [10], [7].

References

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