

**KMR RESEARCH SEMINAR ON GEOMETRIC GROUP THEORY 2014
SCHEDULE**

Wednesday, 26.02.2014

Time	Speaker	Topic
08:30-09:45	Gabi Link	Introduction to cube complexes.
09:45-10:05		Coffee break
10:05-11:20	Olga Varghese	Right-Angled Artin Groups.
11:25-12:40	Robin Loose	Special Cube Complexes I: Introduction.
12:40-13:45		Lunch break
13:45-15:00	Julia Heller and Lukas Buggisch	Special Cube Complexes II: Haglund-Wise Theory.
15:05-16:20	Matthias Nagel	Weak Separation Theorem (Agol-Groves-Manning).

Thursday, 27.02.2014

Time	Speaker	Topic
08:30-09:10		Question hour
09:15-10:30	Cristina Pagliantini	Sageev's cubulation construction.
10:30-11:00		Coffee break
11:00-12:30	Werner Thumann	Overview on 3-manifolds.
12:30-13:45		Lunch break
13:45-15:00	Matthias Blank	Cubulating hyperbolic fundamental groups.
15:00-15:15		Coffee break
15:15-16:30	Gabi Weitze-Schmithüsen	Agol's theorem and the Virtual Haken Conjecture.
17:45		Conference dinner at Fürstliches Brauhaus.

Friday, 28.02.2014

Time	Speaker	Topic
08:30-09:10		Question hour
09:15-10:30	Stefan Friedl	The Virtual Fibration Conjecture.
10:30-11:00		Coffee break
11:00-12:15	Henrik Rüping	The quasi-convex virtual hierarchy.
12:15-13:30		Lunch break
13:30-14:45	Stefan Witzel	Invariant coloring measures.
14:50-16:05	Petra Schwer	Virtually gluing up the hierarchy.

1. SPECIAL CUBE COMPLEXES AND HAGLUND-WISE THEORY

- Talk 1:** Introduction to cube complexes. (Gabi Link)
 Given an overview of cube complexes. In particular, explain the notion of non-positive curvature for cube complexes and Gromov’s link criterion. Hyperplanes. This should cover roughly the material of the first four chapters in Petra’s lecture notes, but can be based on other sources as well.
Literature: For instance: [18], [17], [19].
- Talk 2:** Right-Angled Artin Groups. (Olga Varghese)
 Introduce Coxeter groups and right-angled Artin groups. Examples and basic properties. Explain their relation to cube complexes. RAAGs are linear.
Literature: For instance [10], [11].
- Talk 3:** Special Cube Complexes I: Introduction. (Robin Loose)
 Introduce special cube complexes and some of their properties needed in the proof of Haglund-Wise. Hyperplanes. Cube completion. Examples of special cube complexes. If time permits, relate different notions of “special” [14, Proposition 3.10].
Literature: [14].
- Talk 4:** Special Cube Complexes II: Haglund-Wise Theory. (Julia Heller + Lukas Buggisch)
 Special groups embed into RAAGs: [14, Theorem 4.4]. The Separability Theorem [14, Theorem 7.3] (in detail), [14, Theorem 8.13 and 8.14].
Literature: [14].
- Talk 5:** Weak Separation Theorem (Agol-Groves-Manning). (Matthias Nagel)
 Present the Weak Separation Theorem [1, Theorem A.1], and give an overview of the proof.
Literature: [1, Appendix A], [3].

2. 3-MANIFOLDS AND CUBULATIONS

- Talk 6:** Sageev’s cubulation construction. (Cristina Pagliantini)
 Codimension 1-subgroups. Sageev’s cubulation construction [15, Proposition 3.1], in detail. Examples of the construction. Cocompactness in the hyperbolic case [16, Proposition 3.1].
Literature: [15], [16], [7].
- Talk 7:** Overview on 3-manifolds. (Werner Thumann)
 Explain basic properties of 3-manifolds. The Prime Decomposition Theorem and the Perelman-Thurston Geometrisation Theorem. Examples. Separability and lifting of immersed surfaces. This talk will be 75 + 15 minutes.
Literature: [4]
- Talk 8:** Cubulating hyperbolic fundamental groups. (Matthias Blank)
 Give a short overview of the work of Kahn-Markovic and explain the main result [12, Theorem 1.1] [6, Proposition 5.1]. Discuss in detail the boundary criterion for cubulation [6, Theorem 1.4], and the corollary that fundamental groups of hyperbolic manifolds are cubulated [6, Theorem 5.3].
Literature: [5], [6], [12].

3. AGOL’S THEOREM AND APPLICATIONS

- Talk 9:** Agol’s theorem, the Virtual Haken Conjecture and other applications. (Gabi Weitze-Schmithüsen)
 Present Agol’s theorem and give a very short overview of the proof. Prove the Virtual Haken Conjecture. Give some applications of this conjecture. If time permits, indicate other applications to Geometric Group Theory and Geometry.
Literature: [1], [2], [9, Section 8], [7].
- Talk 10:** The Virtual Fibration Conjecture. (Stefan Friedl)
 Explain and prove Agol’s Virtual Fibration Criterion. Deduce the Virtual Fibration Conjecture.
Literature: [2], [13]
- Talk 11:** Proof of Agol’s Theorem I: The quasi-convex virtual hierarchy. (Henrik Rüping)
 Introduce the fundamental definitions for the proof, including the notion of quasi-convex virtual hierarchy. Virtual Gluing. Quotients with compact walls.
Literature:[1, Section 2, 3, 4], [10], [7].
- Talk 12:** Proof of Agol’s Theorem II: Invariant coloring measures. (Stefan Witzel)
 Prove [1, Theorem 5.2]. Discuss cube complexes with boundary patterns.
Literature: [1, Section 5, 6], [10].
- Talk 13:** Proof of Agol’s Theorem III: Virtually gluing up the hierarchy. (Petra Schwer)
 Finish the proof of Agol’s Theorem.
Literature: [1, Section 8], [10], [7].

REFERENCES

- [1] I. Agol, *The virtual Haken conjecture With an appendix by Agol, Daniel Groves, and Jason Manning*, Doc. Math. 18, 1045-1087 (2013).
- [2] I. Agol, *Criteria for virtual fibering*, J. Topol. 1, no. 2, 269-284 (2008).
- [3] I. Agol, D. Groves, J. Manning *Residual finiteness, QCERF, and fillings of hyperbolic groups*, preprint, arXiv:0802.0709 (2008).
- [4] M. Aschenbrenner, S. Friedl, H. Wilton, *3-manifold groups*, preprint, arXiv:1205.0202.
- [5] N. Bergeron, *La conjecture des sous-groupes de surfaces (d'après Jeremy Kahn et Vladimir Markovic)*, Séminaire Bourbaki, 2011-12, no 1055.
- [6] N. Bergeron, D. Wise, *A boundary criterion for cubulation*, Amer. J. Math. 134, no. 3, 843-859 (2012).
- [7] M. Bestvina, *Geometric group theory and 3-manifolds hand in hand: the fulfillment of Thurston's vision*. Bull. Amer. Math. Soc. 51, no. 1, 53-70 (2014).
- [8] M. Bridson, A. Haefliger, *Metric spaces of non-positive curvature*, Grundlehren der Mathematischen Wissenschaften 319. Springer-Verlag, Berlin, 1999
- [9] D. Calegari, *Notes on Agol's Virtual Haken Theorem*, Lecture notes, Chicago (2013). math.uchicago.edu/~dannyc/courses/agol_virtual_haken/agol_notes.pdf
- [10] R. Charney, *An introduction to right-angled Artin groups*, Geom. Dedicata 125, 141-158 (2007).
- [11] M. Davis, *The geometry and topology of Coxeter groups*, London Mathematical Society Monographs Series, 32. Princeton University Press, Princeton, New Jersey (2008).
- [12] N. J. Kahn, V. Markovic, *Immersing almost geodesic surfaces in a closed hyperbolic three manifold*, Ann. of Math. (2) 175, no. 3, 1127-1190 (2012).
- [13] S. Friedl and T. Kitayama, *The virtual fibering theorem for 3-manifolds*, preprint, arXiv:1210.4799.
- [14] F. Haglund, D. Wise, *Special cube complexes*, Geom. Funct. Anal. 17, no. 5, 1551-1620 (2008).
- [15] M. Sageev, *Ends of group pairs and non-positively curved cube complexes*, Proc. London Math. Soc. 71, no. 3, 585-617 (1995).
- [16] M. Sageev, *Codimension-1 subgroups and splittings of groups*, J. Algebra 189, no. 2, 377-389 (1997).
- [17] M. Sageev, *CAT(0) Cube Complexes and Groups*, lecture notes, PSM12, (2012). www.math.utah.edu/pcmi12/lecture_notes/sageev.pdf
- [18] P. Schwer, *Lecture notes on CAT(0) cube complexes*, Lecture notes, Münster (2013). wwwmath.uni-muenster.de/u/hitzelb/CubeCplx-Lecturenotes.pdf
- [19] D. Wise, *From riches to raags: 3-manifolds, right-angled Artin groups, and cubical geometry*, CBMS Regional Conference Series in Mathematics, 117. Published for the Conference Board of the Mathematical Sciences, Washington; AMS, (2012).