

Seminar: Stable Commutator Length and Quasi-morphisms

Matthias Blank, Stefan Friedl

Summer Term 2015*

Overview

Talk 1 Stable Commutator Length and Surfaces (Daniel Fauser, 14.04.2015)

Surfaces and homology, surfaces and homotopy groups, definition of (stable) commutator length, elementary properties of stable commutator length, stable commutator length and surfaces.

Literature: [3, Sections 1.1.2, 1.1.5, 2.1].

Talk 2 Quasi-morphisms (Johanna Meumertzheim, 28.04.2015)

Definition of quasi-morphisms, homogeneous quasi-morphisms and homogenisation, Bavard's Lemma, examples: de Rham quasi-morphisms, counting quasi-morphisms, rotation number.

Literature: [3, 5, Section 2.2].

Talk 3 Bounded Cohomology and Semi-norms (Enrico Toffoli, 05.05.2015)

Definition of bounded cohomology, bounded cohomology and amenable groups (sketch of proof), Gromov norm and duality principle, Thurston norm and Theorem of Gabai (without proof), definition of simplicial volume and theorem of Gromov and Thurston ([6, Theorem 2.9.7]) (sketch of the proof of the easy implication).

Literature: [3, Sections 2.4.1, 2.4.2, 1.2.5, 4.1],[6, Section 2.9, 2.6.5, 2.5.1, 2.5.2, Proposition 2.3.13].

Talk 4 Bounded Cohomology and Quasi-morphisms (Thomas Markl, 12.05.2015)

Bounded cohomology and quasi-morphisms, Gersten norm, defect norm, quasi-morphisms of free groups, Milnor-Wood inequality.

Literature: [3, Section 2.4.3] [6, Section 2.5.3, 2.5.4]

Talk 5 Bavard's Duality (Gerrit Herrmann, 19.05.2015)

Bavard's duality, stable commutator length as a norm, generalised Bavard's duality, extremal quasi-morphisms.

Literature: [3, Section 2.4, 2.5, 2.6, 2.7.1]

*Tuesday 16:00-18:00, M102

Talk 6 Hyperbolic Manifolds I: Introduction (Matthias Nagel, 09.06.2015)

Recall the definition of hyperbolic manifolds, Drilling and Filling, Dehn surgery, Margulis' Lemma, pleated surfaces.

Literature: [3, Section 3.1], [1, Chapter B, Beginning of Section E.4].

Talk 7 Hyperbolic Manifolds II: Length Inequality and Spectral Gap Theorem (TBA, 16.06.2015)

Length Inequality Theorem, Spectral Gap Theorem.

Literature: [3, Section 3.2].

Talk 8 Quasi-morphisms in Hyperbolic Groups (Johannes Prem, 23.06.2015)

Give a sketch of the proof of the theorem of Epstein and Fujiwara.

Literature: [4]

Talk 9 Acylindrically Hyperbolic Groups I: Introduction (Stefan Friedl, 30.06.2015)

Explain the various definitions of acylindrically hyperbolic groups. Give a sketch of the proof that these definitions are equivalent (Theorem 1.2), examples of acylindrically hyperbolic groups ($\text{Out}(F_n)$, mapping class groups, without proof).

Literature: [7]

Talk 10 Acylindrically Hyperbolic Groups II: Quasi-morphisms (Matthias Blank, 07.07.2015)

Theorem 1 (sketch of proof), Proposition 6 (sketch of proof), Theorem 7.

Literature: [2].

References

- [1] R. Benedetti and C. Petronio. *Lectures on hyperbolic geometry*. Universitext. Springer-Verlag, Berlin, 1992.
- [2] M. Bestvina and K. Fujiwara. Quasi-homomorphisms on mapping class groups. *Glas. Mat. Ser. III*, 42(62)(1):213–236, 2007.
- [3] D. Calegari. *scl*, volume 20 of *MSJ Memoirs*. Mathematical Society of Japan, Tokyo, 2009.
- [4] D. Epstein and K. Fujiwara. The second bounded cohomology of word-hyperbolic groups. *Topology*, 36(6):1275–1289, 1997.
- [5] D. Kotschick. What is... a quasi-morphism? *Notices Amer. Math. Soc.*, 51(2):208–209, 2004.
- [6] C. Löh. Group cohomology and bounded cohomology. Lecture notes, Regensburg. Available online via www.mathematik.uni-regensburg.de/loeh/teaching/topologie3_ws0910/prelim.pdf, 2010.
- [7] D. Osin. Acylindrically hyperbolic groups. *ArXiv e-prints:1304.1246*, April 2013.